Relativistic Particles and Commutator Algebras

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Abstract

We construct the vector fields associated to the space-time invariances of relativistic particle theory in flat Euclidean space-time. We show that the vector fields associated to the massive theory give rise to a differential operator realization of the Poincare algebra, while the vector fields associated to the massless theory, including the space-time supersymmetric sector, allow extensions of the conformal algebra in terms of commutators.

1 Introduction

The relativistic particle continues to be one of the most interesting dynamical systems to investigate if one wishes to try to understand aspects of physics at a fundamental level. Among the many reasons for this, we point out that relativistic particle theory has many features that have higher-dimensional analogues in relativistic string theory while, at the same time, is a prototype of general relativity. In the past few time there has been an intense activity in trying to explain a small positive measured value for the cosmological constant [1]. As a consequence of this activity, a number of quite different and interesting analyses of the cosmological constant problem have appeared [2]. In this context, relativistic particle theory may be used as the simplest possible model for the study of the cosmological constant problem because in the "einbein" version the relativistic particle action defines a one-dimensional generally covariant field theory in which the particle's mass may be viewed as a one-dimensional cosmological constant.

On the other side, one of the most interesting aspects of particle theory is its connection with string theory. In the massless limit, the bosonic particle action has a space-time supersymmetric extension [3] that exhibits a local fermionic symmetry [4] which is crucial for the consistency of the on-shell theory because it induces a perfect matching between the bosonic and fermionic degrees of freedom. This space-time supersymmetric massless particle action is

then dimensionally extended to give rise to the Green-Schwarz superstring [3]. The superstring action can then be further extended to supermembranes [5]. However, while in particle theory the existence of this local fermionic symmetry is a natural consequence of the particle's own dynamics, in superstring and supermembrane theories additional Wess-Zumino terms must be included in the action for the fermionic symmetry to be present. This procedure of including Wess-Zumino terms is thus quite artificial and so the origin of this fundamental fermionic invariance, as a manifestation of the system's own dynamics, can be appreciated only in the case of particle theory. This is one of the interests of this work.

Another interesting point of particle theory, that attracted very little attention, is the question if there exists a specific dimension for the consistency of the supersymmetric theory. It is well known that quantum superstrings seem to work only in space-time dimension $D = 10$ while for supermembranes the critical dimension seems to be $D = 11$. However, no restriction on the space-time dimension appears to exist in the case of the superparticle. To parallel the treatment of superstrings, superparticle. theories are then usually formulated in $D = 10$. In this work we study this problem at the classical level only but we are able to give evidence that superparticle theory is a consistent theory only in $D = 9$.

We are also interested here, as a way to gain some insight into the cosmological constant problem, in the relationship between the presence of a non-vanishing mass value in the bosonic particle action and the space-time invariances of the action. We study this problem by associating a space-time vector field to each of the space-time invariances of the action and then computing the algebra defined by the generators of these vector fields. The result we find is that the massless particle action has a larger set of space-time invariances and then the appearance of a non-vanishing mass may be associated to the breaking of some of these extra invariances. A positive non-vanishing value for the cosmological constant could then also be the result of symmetry-breaking mechanisms.

The paper is divided as follows: In section two we review the concept of space-time vector fields and show how the generators of certain specific vector fields give differential operator realizations of the Poincaré algebra and of the conformal algebra in D bigger than 2. In section three we construct the vector field associated to the invariances of the massive particle action in flat space-time and find that its generators give a differential operator realization of the Poincaré algebra. Section four deals with the massless particle action. We find that its invariances lead to a differential operator realization of the conformal algebra. In particular, we show that when the condition for free motion is satisfied, a new invariance of the action permits the construction of an extension of the conformal algebra. In section five we show how the features found in bosonic particle theory may be extended to relativistic particles with space-time supersymmetry. We present our conclusions in section six.

2 Space-time vector fields

Consider a Riemannian manifold with a metric of Euclidean signature

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu \tag{1}$$

Under an infinitesimal coordinate transformation

$$x^\mu \to x^\mu + \delta x^\mu \tag{2}$$

we have

$$\delta g_{\mu\nu} = -(D_\mu \delta x_\nu + D_\nu \delta x_\mu) \tag{3}$$

where

$$D_\mu \delta x_\nu = \partial_\mu \delta x_\nu - \Gamma^\lambda_{\mu\nu} \delta x_\lambda \tag{4}$$

and

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\delta} (\partial_\mu g_{\delta\nu} + \partial_\nu g_{\mu\delta} - \partial_\delta g_{\mu\nu}) \tag{5}$$

Now let us consider the vector field

$$\hat{R}(\xi) = \xi^\mu \partial_\mu \tag{6}$$

such that

$$D_\mu \xi_\nu + D_\nu \xi_\mu = 0 \tag{7}$$

The vector field $\hat{R}(\xi)$ gives rise to a coordinate transformation

$$\delta x^\mu = \hat{R}(\xi) x^\mu = \xi^\mu \tag{8}$$

under which

$$\delta g_{\mu\nu} = -(D_\mu \xi_\nu + D_\nu \xi_\mu) = 0 \tag{9}$$

The vector field $\hat{R}(\xi)$ is known as the Killing vector field and generates isometries of the manifold. ξ^μ is known as the Killing vector. By definition, isometries are transformations which leave the metric invariant and consequently correspond to transformations preserving the length. The algebra of the Killing vector fields generates the algebra of such symmetry transformations.

Now we restrict the discussion to the two special cases that are necessary to understand the results of this work. The first case is that of Euclidean flat space-time. This case is defined by the equations

$$g_{\mu\nu} = \delta_{\mu\nu} \tag{10a}$$

$$\Gamma^\lambda_{\mu\nu} = 0 \tag{10b}$$

The Killing equation in this case becomes

$$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = 0 \tag{11}$$

the solution of which is [6]

$$\delta x_\mu = \xi_\mu = a_\mu + \omega_{\mu\nu} x^\nu \tag{12}$$

where

$$a_\mu = cons\tan t \tag{13a}$$

$$\omega_{\mu\nu} = -\omega_{\nu\mu} = cons\tan t \tag{13b}$$

The Killing vector field in the case of a flat space-time then takes the form

$$\begin{aligned}\hat{R}(\xi) &= \xi^\mu \partial_\mu \\ &= a^\mu \partial_\mu + \omega^{\mu\nu} x_\nu \partial_\mu \\ &= a^\mu P_\mu - \frac{1}{2}\omega^{\mu\nu} M_{\mu\nu}\end{aligned} \tag{14}$$

where we have defined

$$P_\mu = \partial_\mu \tag{15a}$$

$$M_{\mu\nu} = x_\mu \partial_\nu - x_\nu \partial_\mu \tag{15b}$$

P_μ generates space-time translations and $M_{\mu\nu}$ generates space-time rotations. The algebra of these generators is given by

$$[P_\mu, P_\nu] = 0 \tag{16}$$

$$[P_\mu, M_{\nu\lambda}] = \delta_{\mu\nu} P_\lambda - \delta_{\mu\lambda} P_\nu \tag{17}$$

$$[M_{\mu\nu}, M_{\lambda\rho}] = \delta_{\nu\lambda} M_{\mu\rho} + \delta_{\mu\rho} M_{\nu\lambda} - \delta_{\nu\rho} M_{\mu\lambda} - \delta_{\mu\lambda} M_{\nu\rho} \tag{18}$$

where the brackets denote commutators. This is the Poincaré space-time algebra in a commutator version and so the Killing field provides a differential operator realization of the generators of the Poincaré algebra.

The second case of interest here is the case of the conformal Killing vectors. Consider now the vector field

$$\hat{R}(\epsilon) = \epsilon^\mu \partial_\mu \tag{19}$$

such that

$$D_\mu \epsilon_\nu + D_\nu \epsilon_\mu = \frac{2}{d} g_{\mu\nu} D.\epsilon \tag{20}$$

Here d is the number of space-time dimensions and the factor $\frac{2}{d}$ on the right is needed for consistency.

For the flat space-time defined by equations (10) the conformal Killing equation (20) becomes

$$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu = \frac{2}{d} \delta_{\mu\nu} \partial.\epsilon \tag{21}$$

and one can show that the most general solution for this equation for d greater than 2 is [6]

$$\delta x^\mu = \epsilon^\mu = a^\mu + \omega^{\mu\nu} x_\nu + \lambda x^\mu + (2x^\mu x^\nu - \delta^{\mu\nu} x^2) b_\nu \tag{22}$$

The vector field $\hat{R}(\epsilon)$ for the solution (22) is then given by

$$\hat{R}(\epsilon) = a^\mu P_\mu - \frac{1}{2}\omega^{\mu\nu} M_{\mu\nu} + \lambda D + b^\mu K_\mu \tag{23}$$

where

$$D = x^\mu \partial_\mu \tag{24}$$

and

$$K_\mu = (2x_\mu x^\nu - \delta_\mu^\nu x^2)\partial_\nu \tag{25}$$

The two new additional vector fields (24) and (25) on the right of equation (23) are respectively associated with dilatations and conformal boosts. The generators of the vector field $\hat{R}(\epsilon)$ obey the commutator algebra

$$[P_\mu, P_\nu] = 0 \tag{26a}$$

$$[P_\mu, M_{\nu\lambda}] = \delta_{\mu\nu} P_\lambda - \delta_{\mu\lambda} P_\nu \tag{26b}$$

$$[M_{\mu\nu}, M_{\lambda\rho}] = \delta_{\nu\lambda} M_{\mu\rho} + \delta_{\mu\rho} M_{\nu\lambda} - \delta_{\nu\rho} M_{\mu\lambda} - \delta_{\mu\lambda} M_{\nu\rho} \tag{26c}$$

$$[D, D] = 0 \tag{26d}$$

$$[D, P_\mu] = -P_\mu \tag{26e}$$

$$[D, M_{\mu\nu}] = 0 \tag{26f}$$

$$[D, K_\mu] = K_\mu \tag{26g}$$

$$[P_\mu, K_\nu] = 2(\delta_{\mu\nu} D - M_{\mu\nu}) \tag{26h}$$

$$[M_{\mu\nu}, K_\lambda] = \delta_{\nu\lambda} K_\mu - \delta_{\lambda\mu} K_\nu \tag{26i}$$

$$[K_\mu, K_\nu] = 0 \tag{26j}$$

This is the differential operator realization of the conformal algebra in d greater than 2, the extension of the Poincaré algebra (16-18).

3 Relativistic particles

The "einbein" action for a relativistic particle of mass m in a flat D-dimensional space-time is given by [3]

$$S = \frac{1}{2} \int d\tau (e^{-1} \dot{x}^2 - e m^2) \tag{27}$$

where τ is an arbitrary parameter along the particle's world-line and a dot denotes derivatives with respect to τ. The auxiliary coordinate $e(\tau)$ can be identified as the square root of a one-dimensional metric [3] and so action (27) defines a generally covariant one-dimensional field theory where the particle mass m plays the role of a cosmological constant [7]. The great advantage of action (27) is that it has a smooth transition to the $m = 0$ limit.

The classical equations of motion that follow from Hamilton's principle applied to action (27) are

$$\frac{1}{2} e^{-2} \dot{x}^2 = -\frac{1}{2} m^2 \tag{28}$$

$$\frac{d}{d\tau} (e^{-1} \dot{x}^\mu) = 0 \tag{29}$$

Equation (29) shows that the particle will obey the free motion equation

$$\ddot{x}^\mu = 0 \tag{30}$$

only when the condition $\frac{de}{d\tau} = 0$ is satisfied. As the variable $e(\tau)$ may be associated to the geometry of the particle world-line, we may interpret this condition as the condition for a constant, non-dynamical, geometry. The canonical momentum conjugate to x^μ is given by

$$p_\mu = e^{-1} \dot{x}_\mu \tag{31}$$

and so the equation of motion (28) gives rise to the Hamiltonian constraint [8]

$$\phi = \frac{1}{2} (p^2 + m^2) \approx 0 \tag{32}$$

In this work we use Dirac's convention that a Hamiltonian constraint is set equal to zero only after all calculations have been performed. In this sense equation (32) means that ϕ "weakly" vanishes. The presence of the first-class [8] constraint ϕ means that one of the D x^μ is an unobservable coordinate. Only $D - 1$ of the D x^μ are physical coordinates.

Let us study the space-time invariances of action (27). It is invariant under Poincaré transformations

$$\delta x^\mu = a^\mu + \omega^\mu_\nu x^\nu \tag{33a}$$

$$\delta e = 0 \tag{33b}$$

and under the diffeomorphisms

$$\delta x^{\mu} = \dot{\epsilon} x^{\mu} \tag{34a}$$

$$\delta e = \frac{d}{d\tau}(\epsilon e) \tag{34b}$$

In consequence of the invariance of action (27) under the Poincaré transformation (33) we know that the following vector field can be defined on space-time

$$V = a^{\mu} P_{\mu} - \frac{1}{2}\omega^{\mu\nu} M_{\mu\nu} \tag{35}$$

As we saw in the previous section, the generators of this field provide a differential operator realization of the Poincaré algebra (16-18).

4 Massless relativistic particles

Let us now consider the massless particle action

$$S = \frac{1}{2}\int d\tau e^{-1}\dot{x}^2 \tag{36}$$

which is the $m = 0$ limit of action (27). This action is also invariant under the Poincaré transformations (33) and under the diffeomorphisms (34). The equations of motion that follow from (36) are

$$\frac{1}{2}e^{-2}\dot{x}^2 = 0 \tag{37}$$

$$\frac{d}{d\tau}(e^{-1}\dot{x}^{\mu}) = 0 \tag{38}$$

and as equation (38) indicates, the massless particle will only be free, with a motion described by eq. (30), if the condition $\frac{de}{d\tau} = 0$ is satisfied. If we compute the canonical momentum conjugate to x^{μ} that follows from actin (36) we get

$$p^{\mu} = e^{-1}\dot{x}^{\mu} \tag{39}$$

and so equation (37) is equivalent to the first-class Hamiltonian constraint

$$\phi = \frac{1}{2}p^2 \approx 0 \tag{40}$$

Again, only $D - 1$ of the x^{μ} are physical. As we will see in the next section, the supersymmetric extension of constraint ϕ will not only reduce the bosonic degrees of freedom but will also create the condition for the appearance of the local fermionic symmetry.

The massless action (36) has a larger set of space-time invariances than the massive action (27). It is also invariant under the scale transformation

$$\delta x^\mu = \lambda x^\mu \tag{41a}$$

$$\delta e = 2\lambda e \tag{41b}$$

where λ is a constant, and under the conformal transformation

$$\delta x^\mu = (2x^\mu x^\nu - \delta^{\mu\nu} x^2) b_\nu \tag{42a}$$

$$\delta e = 4ex.b \tag{42b}$$

where b_μ is a constant vector. As a consequence of the invariances of the massless action we can define, according to equation (22), the space-time vector field

$$V_0 = a^\mu P_\mu - \frac{1}{2}\omega^{\mu\nu} M_{\mu\nu} + \lambda D + b^\mu K_\mu \tag{43}$$

As we saw in section two, the generators of the vector field V_0 provide a differential operator realization of the conformal algebra.

Let us now restrict the analysis to the case of free motion. Using equation (30), it can be verified that the massless particle action (36) is also invariant under the transformation

$$x^\mu \rightarrow \exp\{\frac{1}{3}\beta(\dot{x}^2)\} x^\mu \tag{44a}$$

$$e \rightarrow \exp\{\frac{2}{3}\beta(\dot{x}^2)\} e \tag{44b}$$

where β is an arbitrary function of $\dot{x}^2$. This symmetry is interesting because it has a higher-dimensional extension in the tensionless limit of bosonic string theory [9] and, as the results of the next section indicate, it may also be present in the tensionless limit of superstring theory. Just as the massless limit is the high-energy limit of particle theory, the tensionless limit [10] is the high-energy limit [11] of string theory. Under the conditions required for the appearance of invariance (44), we may say that the Hamiltonian constraint ϕ of equation (40) is the generator of this invariance.

Invariance of the massless action under transformation (44) means that infinitesimally we can define the scale transformation

$$\delta x^\mu = \lambda \beta(\dot{x}^2) x^\mu \tag{45}$$

where λ is the same constant that appears in equations (22) and (23). These transformations then lead to the existence of a new type of dilatations. These

new dilatations manifest themselves in the fact that the vector field D of equation (24) can be changed according to

$$\begin{aligned} D &= x^\mu \partial_\mu \to D^* = x^\mu \partial_\mu + \beta(\dot{x}^2) x^\mu \partial_\mu \\ &= D + \beta D \end{aligned} \tag{46}$$

In fact, because all vector fields in equation (43) involve partial derivatives with respect to x^μ , and β is a function of $\dot{x}^\mu$ only, we can also introduce the generators

$$P_\mu^* = P_\mu + \beta P_\mu \tag{47}$$

$$M_{\mu\nu}^* = M_{\mu\nu} + \beta M_{\mu\nu} \tag{48}$$

$$K_\mu^* = K_\mu + \beta K_\mu \tag{49}$$

and define a new vector field V_0^* by

$$V_0^* = a^\mu P_\mu^* - \frac{1}{2}\omega^{\mu\nu} M_{\mu\nu}^* + \alpha D^* + b^\mu K_\mu^* \tag{50}$$

The generators of this vector field obey the commutator algebra

$$[P_\mu^*, P_\nu^*] = 0 \tag{51a}$$

$$[P_\mu^*, M_{\nu\lambda}^*] = (\delta_{\mu\nu} P_\lambda^* - \delta_{\mu\lambda} P_\nu^*) + \beta(\delta_{\mu\nu} P_\lambda^* - \delta_{\mu\lambda} P_\nu^*) \tag{51b}$$

$$\begin{aligned} [M_{\mu\nu}^*, M_{\lambda\rho}^*] &= (\delta_{\nu\lambda} M_{\mu\rho}^* + \delta_{\mu\rho} M_{\nu\lambda}^* - \delta_{\nu\rho} M_{\mu\lambda}^* - \delta_{\mu\lambda} M_{\nu\rho}^*) \\ &\quad + \beta(\delta_{\nu\lambda} M_{\mu\rho}^* + \delta_{\mu\rho} M_{\nu\lambda}^* - \delta_{\nu\rho} M_{\mu\lambda}^* - \delta_{\mu\lambda} M_{\nu\rho}^*) \end{aligned} \tag{51c}$$

$$[D^*, D^*] = 0 \tag{51d}$$

$$[D^*, P_\mu^*] = -P_\mu^* - \beta P_\mu^* \tag{51e}$$

$$[D^*, M_{\mu\nu}^*] = 0 \tag{51f}$$

$$[D^*, K_\mu^*] = K_\mu^* + \beta K_\mu^* \tag{51g}$$

$$[P_\mu^*, K_\nu^*] = 2(\delta_{\mu\nu} D^* - M_{\mu\nu}^*) + 2\beta(\delta_{\mu\nu} D^* - M_{\mu\nu}^*) \tag{51h}$$

$$[M_{\mu\nu}^*, K_\lambda^*] = (\delta_{\lambda\nu} K_\mu^* - \delta_{\lambda\mu} K_\nu^*) + \beta(\delta_{\lambda\nu} K_\mu^* - \delta_{\lambda\mu} K_\nu^*) \tag{51i}$$

$$[K_\mu^*, K_\nu^*] = 0 \tag{51j}$$

Notice that the vanishing brackets of the conformal algebra (26) are preserved as vanishing in the above algebra, but the non-vanishing brackets of the conformal algebra now have linear and quadratic contributions from the arbitrary function $\beta(\dot{x}^2)$. It is interesting to choose β simply linear in $\dot{x}^2$ because then, if the classical equation of motion for $e(\tau)$ that follows from the massless action (36) is imposed, transformation (44) becomes the identity transformation. The algebra (51) is then on-shell in x^μ but off-shell in e. The algebra (51) is an extension of the conformal algebra (26).

5 Superparticles

Finally we consider the case of relativistic particles with space-time supersymmetry, also called superparticles. The bosonic massless particle action (36) has the space-time supersymmetric extension [3]

$$S = \frac{1}{2}\int d\tau e^{-1}(\dot{x}^\mu - i\bar{\theta}\Gamma^\mu\dot{\theta})^2 \tag{52}$$

Here we choose θ_α to be a 32-component spinor ($\alpha = 1, 2, ..., 32$). Γ^μ are appropriate Dirac matrices and $\bar{\theta} = \theta^\dagger\Gamma^0$. As will become clear in the following, a consistent supersymmetric theory, with an equal number of on-shell physical bosons and fermions, can only be defined if the space-time dimension is $D = 9$. Appropriate spinor and Γ matrix representations in this dimension are known to exist [5].

Introducing an infinitesimal constant Grassmann parameter ε, spinor of the same type as the θ coordinate, supersymmetry is realized in space-time as invariance of action (52) under the transformation [3]

$$\delta x^\mu = i\bar{\varepsilon}\Gamma^\mu\theta \tag{53a}$$

$$\delta\theta = \varepsilon \tag{53b}$$

$$\delta e = 0 \tag{53c}$$

The equations of motion that follow from action (52) are

$$\frac{1}{2}e^{-2}(\dot{x}^\mu - i\bar{\theta}\Gamma^\mu\dot{\theta})^2 = 0 \tag{54}$$

$$\frac{d}{d\tau}(e^{-1}Z^\mu) = 0 \tag{55}$$

$$\frac{d}{d\tau}(e^{-1}\Gamma.Z\theta) = 0 \tag{56}$$

where, following reference [3], we introduced the supersymmetric variable $Z^\mu = \dot{x}^\mu - i\bar{\theta}\Gamma^\mu\dot{\theta}$. From equations (55) and (56) it is clear that the superparticle will obey the free motion equations [3]

$$\frac{dZ^\mu}{d\tau} = \ddot{x}^\mu - i\bar{\theta}\Gamma^\mu\ddot{\theta} = 0 \tag{57}$$

$$\Gamma.Z\dot{\theta} = 0 \tag{58}$$

only when the condition $\frac{de}{d\tau} = 0$ is satisfied.

If we compute the canonical momentum conjugate to x^μ that follows from action (52) we get

$$p^\mu = e^{-1}(\dot{x}^\mu - i\bar{\theta}\Gamma^\mu\dot{\theta}) \tag{59}$$

and so equation (54) gives the Hamiltonian constraint

$$\phi = \frac{1}{2}p^2 \approx 0 \tag{60}$$

The matrix $\Gamma.Z$ satisfies [3]

$$(\Gamma.Z)^2 = -Z^2 = -e^2p^2 = 0 \tag{61}$$

and as a result half of the θ_α components in equation (58) are eliminated from the theory. Only 16 independent components remain. Now action (52) is invariant under the fermionic κ-transformation [4]

$$\delta\theta = i\Gamma.Z\kappa \tag{62a}$$

$$\delta x^\mu = i\bar{\theta}\Gamma^\mu\delta\theta \tag{62b}$$

$$\delta e = 4e(\dot{\bar{\theta}})\kappa \tag{62c}$$

where $\kappa(\tau)$ is a Grassmann parameter. The matrix $\Gamma.Z$ appears again in the transformation equation for θ and invariance of the action under (62) means that half of the θ_α components can be gauged away from the theory. Thus only 8 independent components remain. This is a consequence of the presence of the Hamiltonian constraint ϕ of equation (60) which, as we saw, also reduces the number of bosonic degrees of freedom to be $D-1$. A matching between the bosonic and fermionic degrees of freedom can then only happen for $D=9$. This matching of 8 physical degrees of freedom of each type in superparticle theory also occurs in the $D=10$ superstring [3] and in the $D=11$ supermembrane [5]. These three theories describe the same number of physical degrees of freedom.

The superparticle action (52) is also invariant under the local bosonic transformation [3]

$$\delta\theta = \chi\dot{\theta} \tag{63a}$$

$$\delta x^\mu = i\bar{\theta}\Gamma^\mu\delta\theta \tag{63b}$$

$$\delta e = 0 \tag{63c}$$

where $\chi(\tau)$ is a scalar parameter. Invariances (62) and (63) are the usual gauge invariances of the massless superparticle.

Now we notice that from the Poincare transformation (33a) we have

$$\delta\dot{x}^\mu = \delta(\frac{dx^\mu}{d\tau}) = \frac{d}{d\tau}(\delta x^\mu) = \omega^\mu_\nu \dot{x}^\nu \tag{64}$$

and the covariance of the supersymmetric variable Z^μ under Poincaré transformations requires that

$$\delta(-i\bar{\theta}\Gamma^\mu\dot{\theta}) = \omega^\mu_\nu(-i\bar{\theta}\Gamma^\nu\dot{\theta}) \tag{65}$$

since $-i\bar{\theta}\Gamma^\mu\dot{\theta}$ must behave as a Lorentz vector, in the same way as $\dot{x}^\mu$. We can then define the Lorentz transformation

$$\delta Z^\mu = \omega^\mu_\nu Z^\nu \tag{66a}$$

$$\delta e = 0 \tag{66b}$$

and check that action (52) is invariant under transformation (66). Now we point out that the superparticle action (52) also exhibits invariance under a supersymmetric scale transformation give by

$$\delta Z^\mu = \lambda Z^\mu \tag{67a}$$

$$\delta e = 2\lambda e \tag{67b}$$

and also under the supersymmetric conformal transformation

$$\delta Z^\mu = (2Z^\mu Z^\nu - \delta^{\mu\nu} Z^2)B_\nu \tag{68a}$$

$$\delta e = 2Z.Be \tag{68b}$$

where B_μ is a constant vector, which can be chosen for instance as the value of Z_μ when $\tau = 0$. Our experience with the massless particle leads us to expect that, as a consequence of the invariance of the superparticle action (52) under transformations (66), (67) and (68), the vector field

$$\bar{V}_0 = -\frac{1}{2}\omega^{\mu\nu}\bar{M}_{\mu\nu} + \lambda\bar{D} + B^\mu\bar{K}_\mu \tag{69}$$

may be defined in space-time. The generators of this vector field are given by

$$\bar{M}_{\mu\nu} = Z_\mu\frac{\partial}{\partial Z^\nu} - Z_\nu\frac{\partial}{\partial Z^\mu} \tag{70}$$

$$\bar{D} = Z^{\mu}\frac{\partial}{\partial Z^{\mu}} \tag{71}$$

$$\bar{K}_{\mu} = (2Z_{\mu}Z^{\nu} - \delta^{\nu}_{\mu}Z^{2})\frac{\partial}{\partial Z^{\nu}} \tag{72}$$

These generators obey the algebra

$$[\bar{M}_{\mu\nu}, \bar{M}_{\lambda\rho}] = \delta_{\nu\lambda}\bar{M}_{\mu\rho} + \delta_{\mu\rho}\bar{M}_{\nu\lambda} - \delta_{\nu\rho}\bar{M}_{\mu\lambda} - \delta_{\mu\lambda}\bar{M}_{\nu\rho} \tag{73a}$$

$$[\bar{D}, \bar{D}] = 0 \tag{73b}$$

$$[\bar{D}, \bar{M}_{\mu\nu}] = 0 \tag{73c}$$

$$[\bar{D}, \bar{K}_{\mu}] = \bar{K}_{\mu} \tag{73d}$$

$$[\bar{M}_{\mu\nu}, \bar{K}_{\lambda}] = \delta_{\nu\lambda}\bar{K}_{\mu} - \delta_{\lambda\mu}K_{\nu} \tag{73e}$$

$$[\bar{K}_{\mu}, \bar{K}_{\nu}] = 0 \tag{73f}$$

Because the supersymmetric action (52) is not invariant under translations of the Z^{μ} variable, this algebra is the supersymmetric extension of the bosonic conformal algebra (26).

To conclude this work we point out that when equation (57) for free motion is satisfied the superparticle action (52) is invariant under the supersymmetric extension of the bosonic massless particle transformation (44). The transformation equations are

$$x^{\mu} \rightarrow \exp\{\frac{1}{3}\gamma(Z^{2})\}x^{\mu} \tag{74a}$$

$$\theta \rightarrow \exp\{\frac{1}{6}\gamma(Z^{2})\}\theta \tag{74b}$$

$$e \rightarrow \exp\{\frac{2}{3}\gamma(Z^{2})\}e \tag{74c}$$

where γ is an arbitrary function of Z^{2}. As a consequence of the invariance of action (52) under this transformation, we may extend the scale transformations (67a) to transformations of the type

$$\delta Z^{\mu} = \lambda\gamma(Z^{2})Z^{\mu} \tag{75}$$

which in turn leads to the existence of the generator

$$\bar{D}^{*} = \gamma(Z^{2})\bar{D} = \gamma(Z^{2})Z^{\mu}\frac{\partial}{\partial Z^{\mu}} \tag{76}$$

Here we can not proceed as in the bosonic case and introduce also extended operators to $\bar{M}_{\mu\nu}$ and K_μ because these operators involve partial derivatives with respect to Z^μ. The best we can do is define the extended vector field

$$\bar{V}_0^* = -\frac{1}{2}\omega^{\mu\nu}\bar{M}_{\mu\nu} + \lambda(\bar{D} + \bar{D}^*) + B^\mu \bar{K}_\mu \tag{77}$$

and the generators of this vector field obey the algebra (73) complemented with the equations

$$[\bar{D}, \bar{D}^*] = \bar{D}\gamma\bar{D} \tag{78}$$

$$[\bar{M}_{\mu\nu}, \bar{D}^*] = \bar{M}_{\mu\nu}\gamma\bar{D} \tag{79}$$

$$[\bar{K}_\mu, \gamma\bar{D}] = \bar{K}_\mu\gamma\bar{D} - \gamma\bar{K}_\mu \tag{80}$$

6 Conclusions

As the first step, in this work we reviewed the concept of space-time vector fields and how, in the case of Euclidean flat space, these vector fields furnish differential operator realizations of the Poincare and conformal algebra when the space-time dimension is greater than two. Essentially speaking, the origin of these vector fields are space-time transformations. The space-time transformations that leave the relativistic particle action invariant where then used to associate vector fields to the action, and the algebra of the generators of these vector fields was displayed. We saw that the invariances of the massive particle action give rise to a commutator realization of the Poincaré algebra. Taking the limit when the particle's mass goes to zero, we saw that the action becomes more symmetric, being now also invariant under scale and conformal transformations. As a consequence of these extra invariances a larger vector field can be defined in space-time. The generators of this larger vector field were demonstrated to realize a commutator version of the conformal algebra.

An interesting aspect of the free massless theory was shown to be the existence of another type of scale transformation that mixes the dynamical variable x^μ with its derivative with respect to the parameter used to describe the dynamical evolution of the relativistic particle. Transformations that mix the dynamical variables with their derivatives have been classified by some authors as non-internal gauge transformations [12], [13] and we may interpret transformation (44) as one such transformation in the context of generally covariant systems. As we saw, transformation (44), which is present only in the free theory, gives rise to an extension of the conformal algebra.

If we treat the relativistic particle action as a toy model for the study of the cosmological constant problem then the above results allow us to picture the appearance of a particle mass as a sequence of symmetry-breaking mechanisms that induce a transition from the extended conformal algebra (51) down

to the Poincaré algebra (16-18). The first mechanism must induce an interaction. When an interaction appears invariance (44) breaks down and we have a reduction from the extended algebra (51) to the smaller conformal algebra (26). Breakdown of the conformal algebra (26) then reduces the invariances of the action to Poincaré invariance, having as a consequence the appearance of a non-vanishing mass.

As a final analyses in this work we verified if supersymmetric extensions of the algebra (51) could be constructed in the case of massless free superparticles. Although the extension is not straightforward because the action is not invariant under translations of the supersymmetric variable Z^μ, and so the theory is not Poincaré invariant but just Lorentz invariant, we were able to construct a supersymmetric vector field whose generators, defined in terms of the supersymmetric variable Z^μ, partially realize a supersymmetric extension of the commutator algebra (51) for the case of a free massless superparticle. The same symmetry-breaking mechanisms we mentioned in the case of the massless bosonic particle may then also be present in the supersymmetric version of the theory.

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